Pakistan Journal of Engineering and Technology, PakJET
ISSN (p): 2664-2042, ISSN (e): 2664-2050
Volume: 6, Number: 2, Pages: 1-8, Year: 2023

Probability Distribution of Transient Response of an Electric Circuit

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Received: 30/04/2023, Revised: 22/05/2023, Accepted: 24/05/2023

***Abstract*- In this work, we explore the transient response of an electric circuit using probabilistic analysis when the precise value of the characteristic parameter of a circuit element is not known and is instead constrained to a range of values. To accomplish this, a combination of probability theory and traditional circuit analysis techniques is employed. In this approach, parameters of the circuit elements are considered as the random variable. Therefore, probability methods are applied to derive probability distributions across. In this approach, the circuit element's parameter is treated as a random variable, and probability methods are used to derive the probability distribution of the current and across circuit elements. To demonstrate this approach, we apply it to a resistor capacitor (RC) circuit, where the capacitance is regarded as a random variable. We derive probability distributions for the transient voltage across the capacitor and the voltage at the output terminals to illustrate the method's effectiveness.**

***Index Terms*– Circuit probability distribution, RC circuit, transient response, voltage random variable, voltage prediction.**

I. INTRODUCTION

Electric circuits can be analysed using various techniques in either the time or frequency domains. Mesh and nodal analysis, for example, are essential techniques that can be employed in a variety of circuits under varying circumstances [1–4]. These principal techniques underpin several fundamental theorems used in circuit analysis, such as superposition, Thevenin, Norton, and maximum power transfer, which are utilized to analyze different types of circuits [5]. In a continued research on circuit analysis techniques, some recently proposed theorems and algorithms are presented in [6–9].

RC circuits are important in many applications of electrical and electronic engineering and are an active topic of research. Bridged-T and parallel-T RC networks have been investigated in [10] for single resistance control of notch frequencies. It is concluded that bridged-T RC network provides better selectivity compared with parallel-T RC network single resistor control is used. The effect of low pass RC filter on wireless power transfer has been studied in [11]. It is found that the RC circuit affects the radio frequency bandwidth as the harmonics are removed. It also influences the ripple voltage at the rectifier output. An analytical framework using Fourier series is developed to characterize the influence of RC filter on the performance of wireless power transfer. In [12], the authors propose a new structure of RC poly phase filter. The proposed structure uses smaller area and reduces loss without significantly influencing the phase shift performance. The proposed structure is demonstrated by designing a two stage RC poly phase filter which uses 30% smaller area compared with a conventional filter while the loss is reduced by more than 0.6 dB. The work in [13] proposes a method to improve the power utilization efficiency of active RC, continuous time, delta sigma modulator. To improve the power efficiency of the modulator, a large capacitor is introduced in the first amplifier at the virtual ground node. A low pass, low voltage, active RC, fully differential, fourth order Butterworth filter is proposed in [14]. The cutoff frequency is programmable with a resistor bank to four values of 20, 40, 80 and a maximum value of 160 MHz. The RC filter is designed for communication systems operating at the low voltage of 0.6 V. Dynamically activated electrostatic discharge protection is explored in [15]. The work shows that the RC trigger circuit of a high voltage Darlington electrostatic discharge protection affects its performance, and proposes a methodology to evaluate the RC value subject to the constraints of the application.

Deterministic analysis is utilized in all these investigation techniques for electric circuits to achieve a solution as the nominal values of the parameters of circuit elements are used for the analysis. However, certain situations may arise where the circuit element no longer operates within the tolerance specifications of the nominal value and the precise value of a circuit element's characteristic parameter is not available, such as when an element of a circuit malfunctions. The circuit element may operate at values outside the tolerance limits of the nominal value. Deterministic analysis may not be performed under these circumstances as the nominal value of the circuit parameter is not available. Such a situation is analysed in [16], where a probabilistic investigation of a failed resistor capacitor (RC) circuit is performed. The capacitance of a failed capacitor is considered to be in an interval of values. A probability model of the exponentially rising transient response of the RC electric circuit is developed. The analysis of a resistor inductor (RL) circuit is presented in [17] using probability methods. In RL circuit, the value of inductance is partially known and therefore transient response of the RL circuit is investigated by exploiting the probability theory.

To address these scenarios, we propose the use of probability modeling alongside deterministic analysis. Probability theory and modeling are commonly utilized with successful results in various engineering problems, such as noise and channel modeling in electrical communication systems, reliability analysis in civil and mechanical engineering, big data analysis, artificial intelligence and machine learning [18–22]. In this article, we extend and further develop the work in [16] and [17]. We examine the decaying transient response of an RC circuit example in which the capacitance of the capacitor is not precisely known but exists within a range of values. We treat the capacitance as a continuous random variable with uniform distribution and employ conventional analysis techniques to obtain a general expression for the transient response in terms of the voltage across the capacitor in Section II. We subsequently determine the cumulative distribution function and the probability density function of the voltage across the capacitor in Section III. Furthermore, we obtain the cumulative distribution function (CDF) and probability density function (PDF) of the voltage at the circuit's output terminals to demonstrate our approach. The paper concludes with Section IV.

II. TRANSIENT RESPONSE OF RC CIRCUIT

In this section, the transient response of an RC circuit, Figure 1, using conventional circuit analysis techniques is investigated. The circuit includes a capacitor with a capacitance C that is only known to be within a continuous range of values. It is to be noted that the continuous range of values may be well beyond the specified tolerance limits of the nominal value of the capacitance. Consequently, the capacitor no longer operates within the tolerance limits of its nominal value. It should further be noted that the capacitance can be any random value from the continuous interval which remain fix during the transient response. Apart from the capacitor, all other circuit elements operate at their nominal values. We assume that the parasitic resistance and inductance of the capacitor are negligible and are ignored. To demonstrate our probabilistic analysis strategy, we use a numerical example of the circuit shown in Figure 1. The result obtained in this section is then utilized for probabilistic analysis in Section III.

Before opening switch S1 at $t < 0$, we analyze the circuit to determine its initial conditions when the switch is opened at $t = 0$. Circuit is in the steady-state conditions when the switch is closed at $t < 0$. Therefore, at this state, the capacitor is open and charged, as shown in Figure 2. We calculate the currents at node $v_o(0^-)$ to establish the initial conditions.

$$v_o(0^-) = 12\,\text{V}. \tag{1}$$

Now, in Figure 2, at $t < 0$, let the current flowing from 24 V battery to 2 Ω resistor be represented by I_1. The current I_1 is given by,

$$I_1 = 2\,\text{A}. \tag{2}$$

From the left loop in Figure 2 containing the voltage source V1, the 2 Ω resistor and the capacitor with voltage $v_c(0^-)$ at $t < 0$, the sum of voltages yields,

$$v_c(0^-) = 20\,\text{V}. \tag{3}$$

$v_c(0^-)$ is the voltage across the capacitor before the opening of the switch at $t = 0$. As the capacitor voltage does not change

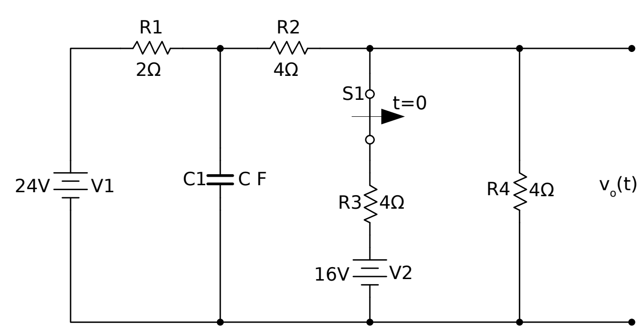

FIGURE 1. RC circuit for the transient response investigation.

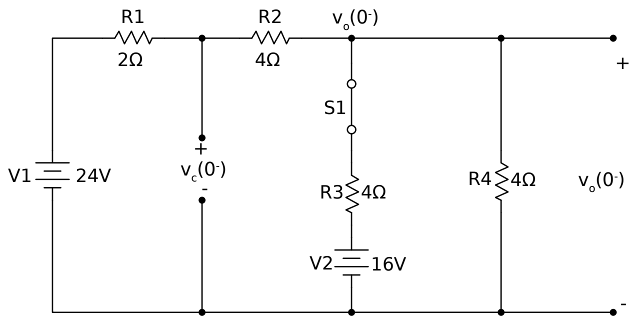

FIGURE 2. At $t < 0$, the circuit shown in the figure is in a steady state where the switch S1 is closed, the capacitor is fully charged, and it is open.

instantaneously, the capacitor voltage after closing the switch at $t = 0^+$ or simply at $t = 0$ remains unchanged. Therefore,

$$v_c(0) = v_c(0^+) = v_c(0^-) = 20 \text{ V}. \tag{4}$$

Let us now analyze the circuit at $t > 0$ after the switch S1 is opened. At $t > 0$, the branch containing the switch S1 is no longer in the circuit as is shown in Figure 3. If the capacitor voltage is represented by v_c, then considering the currents entering and leaving the node at the top of the capacitor in Figure 3, we obtain,

$$8C\frac{dv_c}{dt} + 5v_c = 96. \tag{5}$$

This is first order differential equation whose general solution is given by

$$v_c(t) = k_1 + k_2 e^{-\frac{t}{\tau}}, \tag{6}$$

where k_1, k_2 and τ are constants. τ is the time constant which is a characteristic of the circuit. The constants are found to be:

$$k_1 = \frac{96}{5}, \tag{7}$$

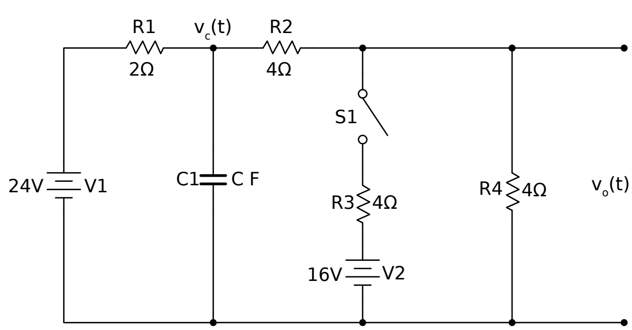

FIGURE 3. At $t > 0$, the switch S1 is opened in the circuit shown in the figure. The branch containing the switch S1 is no longer part of the circuit.

$$k_2 = \frac{4}{5}, \tag{8}$$

$$\tau = \frac{8C}{5} \text{ seconds}. \tag{9}$$

Therefore, from (6),

$$v_c(t) = \frac{96}{5} + \frac{4}{5}e^{-\frac{5}{8C}t}. \tag{10}$$

From (10), we can observe that at $t = 0$ and $t = \infty$, v_c is 20 V and 19.2 V respectively. Therefore, the transient voltage across the capacitor exhibits an exponential decay. We plot this transient response for a convenient value of $C = 4$ F in Figure 4.

From Figure 3, it is evident that resistors R2 and R4 are in series when the switch S1 is open. Therefore, we can obtain $v_o(t)$, as under,

$$v_o(t) = \frac{48}{5} + \frac{2}{5}e^{-\frac{5}{8C}t} \text{ V}. \tag{11}$$

From (11), we get v_o 10 V and 9.6 V at $t = 0$ and $t = \infty$ respectively. Hence, the RC circuit exhibits an exponential decay in its transient response.

III. PROBABILITY DISTRIBUTION

This section is focused on deriving the probability distribution of the transient response using the result obtained in Section II. As mentioned in Section II., the capacitance C has equal probability of taking any value within a continuous interval. In probability theory, this phenomenon is modeled using the continuous uniform distribution. Therefore, C can be treated as a continuous uniform random variable in our problem. Consequently, the capacitor

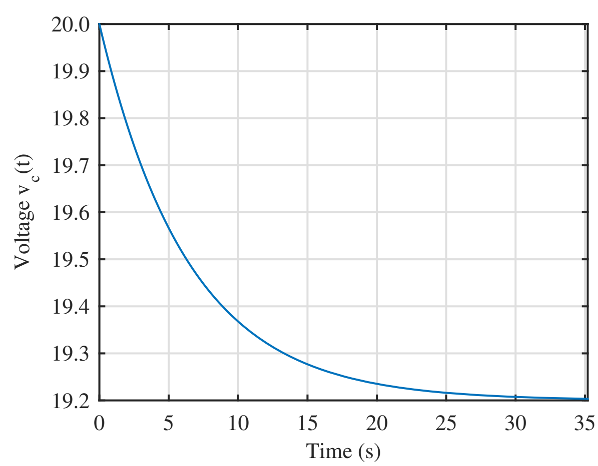

FIGURE 4. As the switch S1 is opened, the voltage across the capacitor decays exponentially from $v_c = 20$ V at $t = 0$ to $v_c = 19.2$ V at $t = \infty$.

voltage is also a random variable that is dependent on the random variable C. It is to be noted that an alternate and appropriate probability distribution can be used to model the capacitance random variable if the nature of the capacitor fault is different such that the values of the capacitance C are not equally probable. The alternate probability distribution may be chosen depending on the nature of the capacitor fault and the information available about it. Throughout this work, random variable and its value is denoted by a capital and lowercase letters respectively. For instance, the value of any random variable x represents any of the possible values that the random variable X can take.

A. CDF OF V_c

Assuming that C lies within a continuous interval bounded by a and b, and it takes any of the values within this interval with equal probability, it can be considered as a continuous random variable uniformly distributed between a and b. In this case, the PDF of C, $f_C(c)$, is given by,

$$f_C(c) = \begin{cases} \frac{1}{b-a} & a \leq c < b, \\ 0 & \text{otherwise,} \end{cases} \tag{12}$$

where both a and b are constants and fulfills the condition that $b > a > 0$. Similarly, the CDF of C, $F_C(c)$, is given by,

$$F_C(c) = \begin{cases} 0 & c \leq a, \\ \frac{c-a}{b-a} & a < c \leq b, \\ 1 & c > b. \end{cases} \tag{13}$$

For $0 < t \leq 5\tau$, the circuit is in the transient state. Let us consider a time within this transient period, given by,

$$t = \frac{n\tau}{C}, \tag{14}$$

where n can be derived as ,

$$\begin{gathered} 0 < t \leq 5\tau, \\ 0 < \frac{n\tau}{C} \leq 5\tau, \\ 0 < n \leq 5C. \end{gathered}$$

From (12) and (13), we know C is a uniform random variable and $a < c \leq b$. Therefore, n in the transient state is,

$$0 < n \leq 5a. \tag{15}$$

Substituting (9) in (14), we obtain,

$$t = \frac{8}{5}n. \tag{16}$$

Using this value in (10), we get the following relationship between C and the derived random variable V_c,

$$V_c = \frac{96}{5} + \frac{4}{5}e^{-\frac{n}{C}}. \tag{17}$$

V_c is a random variable which is derived from C. From (17),

$$C = \frac{-n}{\ln\left[\frac{5}{4}\left(V_c - \frac{96}{5}\right)\right]}. \tag{18}$$

For a uniform(a, b) random variable C, $C > 0$. Therefore, the denominator in (18) fulfills,

$$\ln\left[\frac{5}{4}\left(V_c - \frac{96}{5}\right)\right] < 0, \tag{19}$$

so that we can obtain $C > 0$.

Now, to derive the PDF $f_{V_c}(v_c)$ of V_c, let us first derive the CDF $F_{V_c}(v_c)$, which is given by,

$$F_{V_c}(v_c) = P[V_c \leq v_c]. \tag{20}$$

Using (17), we can rewrite $V_c \leq v_c$, as following,

$$\frac{96}{5} + \frac{4}{5}e^{-\frac{n}{C}} \leq v_c, \tag{21}$$

$$C \leq \frac{-n}{\ln\left[\frac{5}{4}\left(v_c - \frac{96}{5}\right)\right]}. \tag{22}$$

From (20) and (22), we obtain,

$$F_{V_c}(v_c) = P\left[C \leq \frac{-n}{\ln\left[\frac{5}{4}\left(v_c - \frac{96}{5}\right)\right]}\right], \tag{23}$$

$$F_{V_c}(v_c) = F_C\left(\frac{-n}{\ln\left[\frac{5}{4}\left(v_c - \frac{96}{5}\right)\right]}\right). \tag{24}$$

Therefore, from (13), we obtain,

$$\frac{c-a}{b-a} = \frac{-n}{(b-a)\left[\ln\left[\frac{5}{4}\left(v_c - \frac{96}{5}\right)\right]\right]} - \frac{a}{b-a}. \tag{25}$$

To completely determine $F_C(c)$, we need to evaluate three different intervals which correspond to $c \leq a$, $a < c \leq b$ and $c > b$ as can be seen from (13). These are evaluated in the following. For $c \leq a$, using (18), we obtain,

$$\frac{-n}{\ln\left[\frac{5}{4}\left(v_c - \frac{96}{5}\right)\right]} \leq a, \tag{26}$$

$$v_c \leq \frac{96}{5} + \frac{4}{5}e^{-\frac{n}{a}}. \tag{27}$$

Now the expression for interval $c > b$ can be derived by employing (18) ,

$$\frac{-n}{\ln\left[\frac{5}{4}\left(v_c - \frac{96}{5}\right)\right]} > b, \tag{28}$$

$$v_c > \frac{96}{5} + \frac{4}{5}e^{-\frac{n}{b}}. \tag{29}$$

Similarly, expression obtained for $a < c \leq b$ using (18) is,

$$a < \frac{-n}{\ln\left[\frac{5}{4}\left(v_c - \frac{96}{5}\right)\right]} \leq b, \tag{30}$$

$$\frac{96}{5} + \frac{4}{5}e^{-\frac{n}{a}} < v_c \leq \frac{96}{5} + \frac{4}{5}e^{-\frac{n}{b}}. \tag{31}$$

Using (25), (27), (29) and (31), we can write the CDF $F_{V_c}(v_c)$, as below,

$$F_{V_c}(v_c) = \begin{cases} 0 & v_c \leq \frac{96}{5} + \frac{4}{5}e^{-\frac{n}{a}}, \\ \frac{-n}{(b-a)\left[\ln\left[\frac{5}{4}\left(v_c - \frac{96}{5}\right)\right]\right]} - \frac{a}{b-a} & \frac{96}{5} + \frac{4}{5}e^{-\frac{n}{a}} < v_c \leq \frac{96}{5} + \frac{4}{5}e^{-\frac{n}{b}}, \\ 1 & v_c > \frac{96}{5} + \frac{4}{5}e^{-\frac{n}{b}}. \end{cases} \tag{32}$$

B. PDF OF V_c

The PDF can be obtained from the CDF as below,

$$f_{V_c}(v_c) = \frac{\mathrm{d}F_{V_c}}{\mathrm{d}v_c}, \tag{33}$$

$$f_{V_c}(v_c) = \frac{\mathrm{d}}{\mathrm{d}v_c}\left[\frac{-n}{(b-a)\left[\ln\left[\frac{5}{4}\left(v_c - \frac{96}{5}\right)\right]\right]} - \frac{a}{b-a}\right], \tag{34}$$

$$f_{V_c}(v_c) = \frac{n}{(b-a)(v_c - \frac{96}{5})\left[\ln\left[\frac{5}{4}(v_c - \frac{96}{5})\right]\right]^2}. \tag{35}$$

The PDF $f_{V_c}(v_c)$ is, therefore, given by,

$$f_{V_c}(v_c) = \begin{cases} \frac{n}{(b-a)(v_c - \frac{96}{5})\left[\ln\left[\frac{5}{4}(v_c - \frac{96}{5})\right]\right]^2} & \frac{96}{5} + \frac{4}{5}e^{-\frac{n}{a}} < v_c \leq \frac{96}{5} + \frac{4}{5}e^{-\frac{n}{b}}, \\ 0 & \text{otherwise}. \end{cases} \tag{36}$$

This is confirmed to be a valid PDF as the following is found to hold for (36),

$$\int_{-\infty}^{+\infty} f_{V_c}(v_c)\mathrm{d}v_c = 1. \tag{37}$$

C. EXPECTED VALUE OF V_c

The expected value of V_c is calculated as below,

$$\mathbf{E}[V_c] = \int_{-\infty}^{+\infty} v_c f_{V_c}(v_c)\mathrm{d}v_c, \tag{38}$$

$$\mathbf{E}[V_c] = \int_{\frac{96}{5}+\frac{4}{5}e^{-\frac{n}{a}}}^{\frac{96}{5}+\frac{4}{5}e^{-\frac{n}{b}}} \frac{n v_c}{(b-a)(v_c - \frac{96}{5})\left[\ln\left[\frac{5}{4}(v_c - \frac{96}{5})\right]\right]^2}\mathrm{d}v_c. \tag{39}$$

$$\mathbf{E}[V_c] = \frac{n}{b-a} \times \left| \frac{-v_c}{\ln\left[\frac{5}{4}\left(v_c - \frac{96}{5}\right)\right]} + \frac{4}{5}\mathrm{li}\left[\frac{5}{4}\left(v_c - \frac{96}{5}\right)\right]\right|_{\frac{96}{5}+\frac{4}{5}e^{-\frac{n}{a}}}^{\frac{96}{5}+\frac{4}{5}e^{-\frac{n}{b}}}, \tag{40}$$

$$\mathbf{E}[V_c] = \frac{1}{b-a}\left[b\left(\frac{96}{5} + \frac{4}{5}e^{-\frac{n}{b}}\right) - a\left(\frac{96}{5} + \frac{4}{5}e^{-\frac{n}{a}}\right)\right] + \frac{4n}{5(b-a)}\left[\mathrm{Ei}\left(-\frac{n}{b}\right) - \mathrm{Ei}\left(-\frac{n}{a}\right)\right]. \tag{41}$$

D. EXAMPLE FOR V_c

To provide a specific example, consider the case where $a = 3$ and $b = 5$, resulting in an expected value of $E[C] = 4$ F. Substituting these values into (12) and (13), we obtain the following PDF of C for this interval:

$$f_C(c) = \begin{cases} \frac{1}{2} & 3 \leq c < 5, \\ 0 & \text{otherwise}, \end{cases} \tag{42}$$

The corresponding CDF is given by,

$$F_C(c) = \begin{cases} 0 & c \leq 3, \\ \frac{c-3}{2} & 3 < c \leq 5, \\ 1 & c > 5. \end{cases} \tag{43}$$

Taking (15) into consideration, let $n = a = 3$, so that from (16), we get $t = 4.8$ s. From (9), we note that $t = 4.8 < \tau$, and it lies within the transient state given that $3 < c \leq 5$. Using $n = a = 3$ and $b = 5$ in (32), we obtain,

$$F_{V_c}(v_c) = \begin{cases} 0 & v_c \leq 19.494, \\ \frac{-3}{2\ln\left[\frac{5}{4}\left(v_c - \frac{96}{5}\right)\right]} - \frac{3}{2} & 19.494 \leq v_c < 19.639, \\ 1 & v_c > 19.639. \end{cases} \tag{44}$$

Similarly from (36), we get,

$$f_{V_c}(v_c) = \begin{cases} \frac{3}{2(v_c - \frac{96}{5})\left[\ln\left[\frac{5}{4}(v_c - \frac{96}{5})\right]\right]^2} & 19.494 < v_c \leq 19.639, \\ 0 & \text{otherwise.} \end{cases} \tag{45}$$

The CDF and the PDF for this example are plotted in Figure 5 and Figure 6 respectively.

Let us now, for example, calculate $P[V_c \leq 19.60]$. This can be calculated using either the CDF or the PDF. However, it is easier to use the former. Therefore, from (44), we obtain,

$$P[V_c \leq 19.60] = F_{V_c}(19.60) = 0.664.$$

It can be observed from (44) and (45) that $F_{V_c}(v_c) = 0$ and $f_{V_c}(v_c) = 0$ for $v_c < 19.494$. Therefore, $P[V_c \leq 19.60]$ implies $P[19.494 < V_c \leq 19.60]$. In words, it can be stated that the probability that v_c is between 19.494 and 19.60 at $t = 4.8$ s is 66.4%. This also implies that the probability that v_c is between 19.60 and 19.639 at $t = 4.8$ s is 33.6%.

It would be interesting to calculate the probability for some other value of voltage for the specific case under consideration. For example, $P[V_c \leq 19.50]$ is found out to be as follows,

$$P[V_c \leq 19.50] = F_{V_c}(19.50) = 0.0293.$$

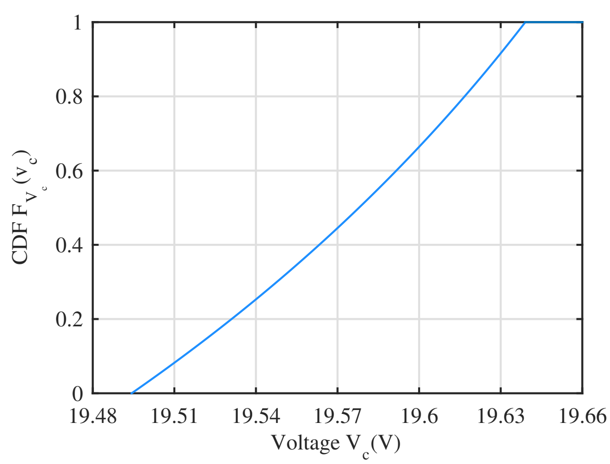

FIGURE 5. The cumulative distribution function (CDF) $F_{V_c}(v_c)$ where C is a uniform random variable between 3 and 5.

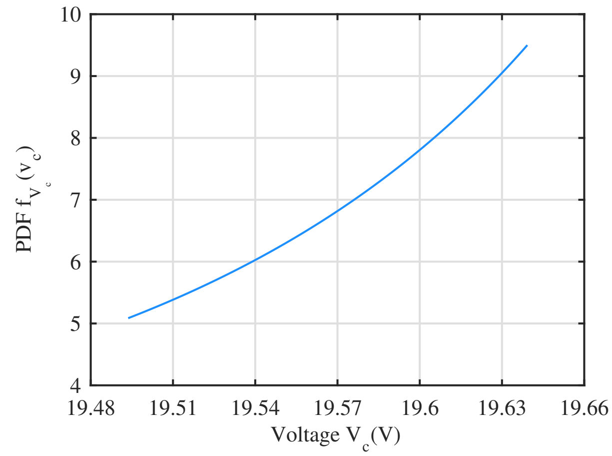

FIGURE 6. The probability density function (PDF) $f_{V_c}(v_c)$ when C is a uniform$(3, 5)$ random variable.

The probability at any other time instant can be calculated by using an appropriate value of n. The probability values for $P[V_c \leq 19.50]$ are calculated for different values of n, and are summarized in Table I.

We can calculate the expected value of V_c also for the particular case when $n = a = 3$ and $b = 5$ as follows,

$$\mathbf{E}[V_c] = \int_{19.494}^{19.639} \frac{3v_c}{2(v_c - \frac{96}{5})\left[\ln\left[\frac{5}{4}(v_c - \frac{96}{5})\right]\right]^2} \mathrm{d}v_c, \tag{46}$$

$$\mathbf{E}[V_c] = 19.595 \text{ V}. \tag{47}$$

E. CDF AND PDF OF V_o

It is worth noting that the output voltage V_o is also a stochastic variable, and its probability density function (PDF) and cumulative distribution function (CDF) can be obtained through a process similar to the one applied for V_c. Alternatively, the CDF $F_{V_o}(v_o)$ and PDF $f_{V_o}(v_o)$ for V_o can be derived from (32) and (36). From (11), we have,

$$V_o = \frac{1}{2} V_c \tag{48}$$

TABLE I

PROBABILITY $P[V_C \leq 19.50]$ FOR DIFFERENT VALUES OF n

n	t	$\mathbf{P[V_c \leq 19.50]}$
3.0	4.8	0.0293
3.5	5.6	0.2842
4.0	6.4	0.5391
4.5	7.2	0.7940

Therefore, V_o is a derived random variable which is a function and constant multiple of V_c. Hence, if $V_o = kV_c$, then

$$F_{V_o}(v_o) = F_{V_c}(\frac{v_o}{k}), \tag{49}$$

$$f_{V_o}(v_o) = \frac{1}{k} f_{V_c}(\frac{v_o}{k}). \tag{50}$$

It can be seen from (48) that $k = \frac{1}{2}$ in this case. Therefore, from (32), (36), (49) and (50), we obtain,

$$F_{V_o}(v_o) = \begin{cases} 0 & v_o \leq \dfrac{48}{5} + \dfrac{2}{5}e^{-\frac{n}{a}}, \\ \dfrac{-n}{(b-a)\left[\ln\left[\frac{5}{2}\left(v_o - \frac{48}{5}\right)\right]\right]} - \frac{a}{b-a} & \dfrac{48}{5} + \dfrac{2}{5}e^{-\frac{n}{a}} < v_c \leq \dfrac{48}{5} + \dfrac{2}{5}e^{-\frac{n}{b}}, \\ 1 & v_c > \dfrac{48}{5} + \dfrac{2}{5}e^{-\frac{n}{b}}. \end{cases} \tag{51}$$

$$f_{V_o}(v_o) = \begin{cases} \dfrac{n}{(b-a)(v_o - \frac{48}{5})\left[\ln\left[\frac{5}{2}(v_o - \frac{48}{5})\right]\right]^2} & \dfrac{48}{5} + \dfrac{2}{5}e^{-\frac{n}{a}} < v_o \leq \dfrac{48}{5} + \dfrac{2}{5}e^{-\frac{n}{b}}, \\ 0 & \text{otherwise.} \end{cases} \tag{52}$$

As an example, for $n = a = 3$ and $b = 5$, we have,

$$F_{V_o}(v_o) = \begin{cases} 0 & v_o \leq 9.747, \\ \dfrac{-3}{2\ln\left[\frac{5}{2}\left(v_o - \frac{48}{5}\right)\right]} - \frac{3}{2} & 9.747 \leq v_o < 9.820, \\ 1 & v_o > 9.820. \end{cases} \tag{53}$$

$$f_{V_o}(v_o) = \begin{cases} \dfrac{3}{2(v_o - \frac{48}{5})\left[\ln\left[\frac{5}{2}(v_o - \frac{48}{5})\right]\right]^2} & 9.747 < v_o \leq 9.820, \\ 0 & \text{otherwise.} \end{cases} \tag{54}$$

As demonstrated in the case of $F_{V_c}(v_c)$ and $f_{V_c}(v_c)$, the CDF and PDF in (53) and (54) can be used for the probabilistic analysis of output voltage v_o.

IV. CONCLUSION

In this article we obtain the cumulative distribution function and probability density function for the voltage across a component in an electric circuit. Specifically, our analysis concentrates on an RC circuit, in which the capacitance of the capacitor is considered a uniformly distributed random variable. By combining conventional analysis with probability theory, we derive the cumulative distribution functions and probability density functions for both the voltage across the capacitor and the voltage at the output terminals.

FUNDING STATEMENT

The authors received no funding for this work.

CONFLICTS OF INTEREST

The authors declare they have no conflicts of interest to report regarding the present work.

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